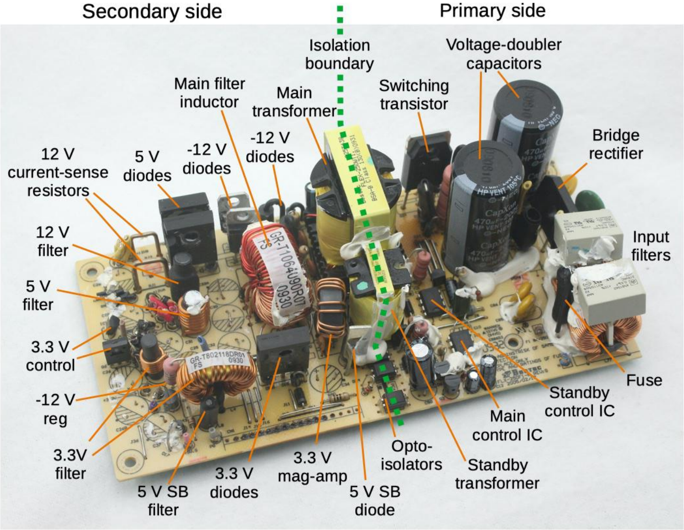
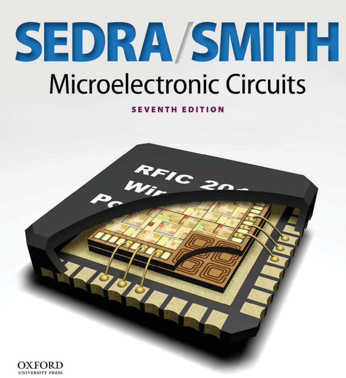


The Small-Signal Model

Kizito NKURIKIYEYEZU, Ph.D.

Readings

- **Section 4.3.7 on page 195-200**
- **Example Example 4.5 on page 198**



[1] Readings are based on Sedra & Smith (2014), Microelectronic Circuits 7th edition.
[2] Bold reading section are mandatory. Other sections are suggested but not required readings

Background

- Previous models have taught us how to find I_D and V_D in Fig. 1

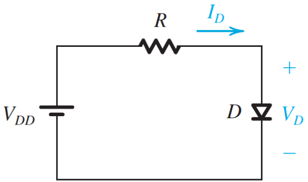


FIG 1. A simple diode circuit

[1] https://en.wikipedia.org/wiki/Small-signal_model
[2] https://en.wikipedia.org/wiki/Diode_modelling#Small-signal_modelling

Background

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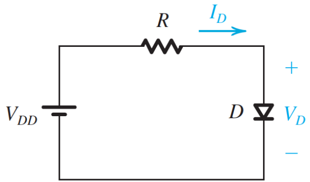
- Now, consider the situation of V_{DD} has a small change ΔV_{DD} (Fig. 2).



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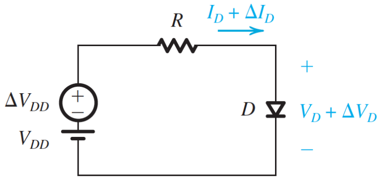

FIG 2. When V_{DD} changes by ΔV_D

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- Previous models have taught us how to find I_D and V_D in Fig. 1
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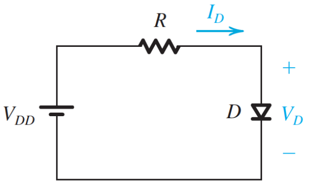

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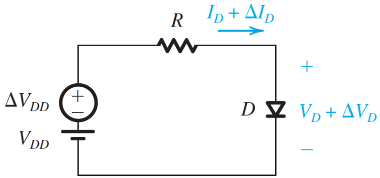

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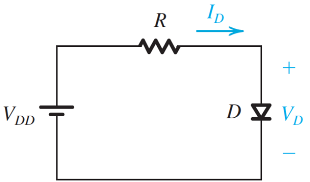
- A small signal model—which uses a Maclaurin series expansion around a specific operating point—helps quickly find these incremental changes



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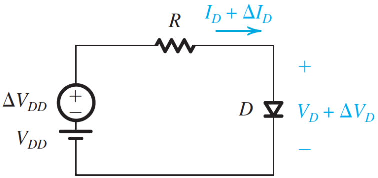

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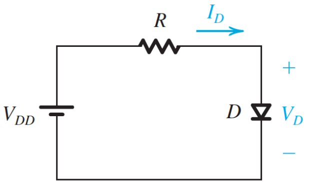
- In this case, the current I_D changes by an increment ΔI_D and the diode voltage V_D changes by ΔV_D
- A small signal model—which uses a Maclaurin series expansion around a specific operating point—helps quickly find these incremental changes
- Note that using a small-signal model fails when the input signal gets too large



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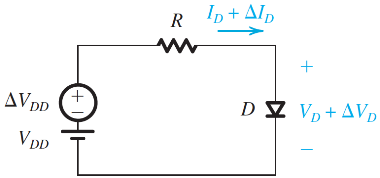

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Small-Signal Model

- A small-signal model for a diode gives you a quick way to analyze nonlinear circuits.
- A diode is modeled as variable resistor.
- Whose value is defined via linearization of exponential model.
- Around bias point defined by constant voltage drop $V_D^{(0)} = 0.7\,V$

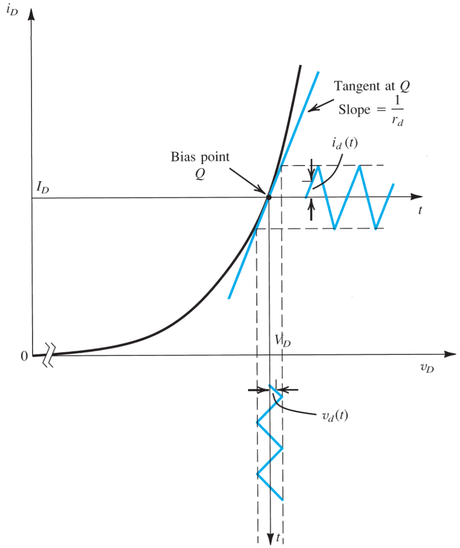

FIG 3. The diode small-signal model.

Small-signal model development

- Develop an equivalent circuit for a diode that is used when a small, time-varying signal is applied to a diode circuit.
- The total instantaneous circuit is divided into steady-state and time varying components, which may be analyzed separately and solved via algebra.
  - In steady-state, diode represented as Constant Voltage Drop Model (CVDM)
  - In time-varying, diode represented as resistor

Small-signal model development

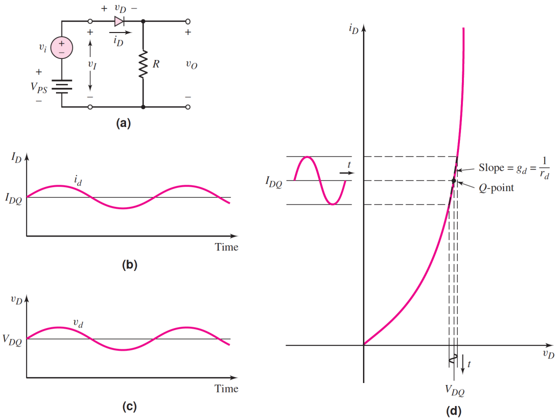

FIG 4. AC circuit analysis: (a) circuit with combined dc and sinusoidal input voltages, (b) sinusoidal diode current superimposed on the quiescent current, (c) sinusoidal diode voltage superimposed on the quiescent value, and (d) forward-biased diode I –V characteristics with a sinusoidal current and voltage superimposed on the quiescent values

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[1] Adapted from Neamen, D. A. (2009). Microelectronics: circuit analysis and design (4th edition). New

Small-Signal Model

- Instantaneous voltage across the diode $V_D(t)$ is a sum of the dc voltage V_D and the time-varying signal $v_d(t)$ (Equation (1))

$$V_D(t) = V_D + v_d(t) \tag{1}$$

where $v_d(t)$ is a small signal voltage compared to V_D

- The corresponding current $i_D(t)$ through the diode is then (Equation (2))

$$i_D(t) = I_s \cdot e^{v_D/V_T} \tag{2}$$

- Substituting Equation (1) in Equation (2) we get

$$\begin{aligned} i_D(t) &= I_s \cdot e^{(v_D + v_d)/V_T} \\ &= I_s \cdot e^{v_D/V_T} \cdot e^{v_d/V_T} \\ &= I_D e^{v_d/V_T} \end{aligned} \tag{3}$$

- Since $v_d/V_T \ll 1$, and since $e^x \approx 1 + x$ when x is small[1], then Equation (3) can be approximated as Equation (4)

$$i_D(t) \approx I_D\left(1 + \frac{v_d(t)}{V_T}\right) = I_D + \frac{I_D}{V_T}v_d(t) \tag{4}$$

- Expressing $i_D(t)$ as Equation (5)

$$i_D(t) = I_D + i_d(t) \tag{5}$$

Where $i_d(t)$ is a small signal current (i.e., $i_d(t) \ll I_D$), then

$$i_d(t) = \frac{I_D}{V_T}v_d(t)^2 \tag{6}$$

- The incremental resistance or the small signal resistance r_d is defined as

$$r_d = \frac{V_T}{I_D} \tag{7}$$

[1] https://en.wikipedia.org/wiki/Taylor_series

[2] The quantity relating the signal current i_d to the signal voltage v_d has the dimensions of conductance, mhos (℧), and is called the diode small-signal conductance.

Small-Signal Model

- Approximations in Equation (4), Equation (5) and Equation (7) converts a nonlinear problem into a linear problem at the DC bias point, the quiescent point, or the Q point.
- This is the small-signal approximation. It is valid for signals whose amplitudes are smaller than about $5mV$

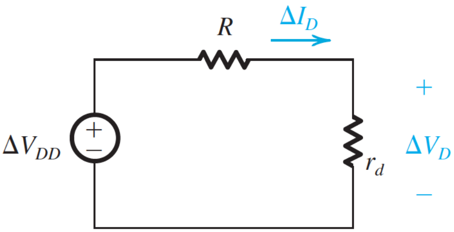
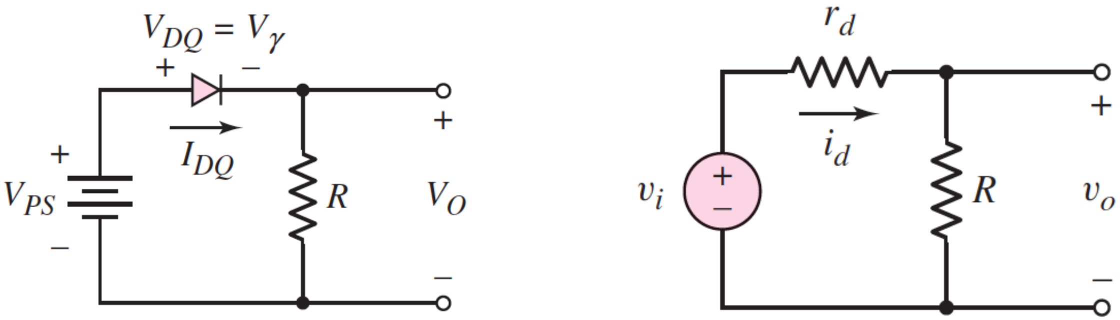


FIG 5. A nonlinear diode circuit can be replaced by a linear resistor circuit under the small signal approximation

Small-Signal Model equivalent



(a) dc equivalent **(b)** ac equivalent

FIG 6. Equivalent circuity of a small signal model

[1] Adopted from Neamen, D. A. (2009). Microelectronics: circuit analysis and design (4th edition). New York: McGraw-Hill.

Mathematical viewpoint

$$\begin{aligned} i_D &= i_D(V_D + v_d) \\ &= i_d(V_D) + v_d \left.\frac{\partial i_d}{\partial v_D}\right|_{v_d=Q} \\ &= I_D + v_D\frac{1}{r_d} \\ &= I_D + i_d(t) \end{aligned} \tag{8}$$

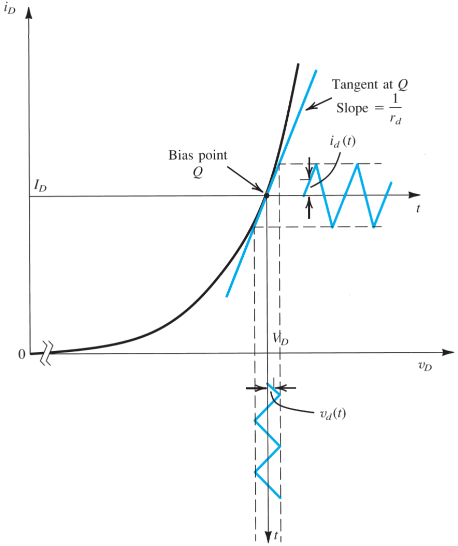


FIG 7. The diode small-signal model